THERMAL BREAKDOWN IN A HEATING FILAMENT

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Circuit components sometimes burn out suddenly, which belongs to a class of critical phenomena covered by macroscopic kinetics [1]. Semenov [2] pointed out the existence of electrical analogs of chemical explosion with reference to thermal breakdown in insulators. A conductor carrying a current differs from an insulator when considered as a localized case of energy release in not having explosive instability. The explosive instability effect arises when one incorporates the distributed nature of the system and the scope for spatial perturbation. Then breakdown can be treated as local evaporation of the conductor because of spontaneous sharpening in the temperature inhomogeneity.

1. We consider a resistance filament thermally insulated at the ends, whose ends have applied a constant potential difference U. The heat-balance equation for the filament in a chemically inert medium is

$$c\frac{\partial T}{\partial t} = \frac{U^2\rho}{r^4\left[\int_0^l \rho r^{-2}\,dx\right]^2} - \frac{2\alpha}{r}(T - T_0) - \frac{2\sigma}{r}T^4 + \frac{1}{r^2}\frac{\partial}{\partial x}\left(r^2\lambda\frac{\partial T}{\partial x}\right), \qquad (1.1)$$

in which x is the longitudinal coordinate, T and T_0 the temperatures of the filament and the surrounding medium, c the specific heat of unit volume, r radius, ℓ length, $\rho(T)$ resistivity, λ thermal conductivity, α heat-transfer coefficient, and σ radiation constant.

When the current source is switched on, the filament's temperature rises rapidly to T_S, which remains stationary if we neglect the slow filament evaporation; T_S is defined by

$$\frac{U^2}{\rho(T_s)\,l^2} = \frac{2\alpha}{r}(T_s - T_0) + \frac{2\sigma}{r}T_s^4. \qquad (1.2)$$

To examine the stability of the homogeneous stationary solution, we examine the evolution of small temperature perturbations. The nonstationary solution T(x, t) is sought as the sum

$$T(x,\,t) = T_s + \theta(t)f(x).$$

We substitute T(x, t) into (1.1) and linearize all the terms to get

$$c\frac{\partial\theta}{\partial t}f(x) = \frac{U^2}{\rho_s l^2}R_T\theta f(x) - \frac{2U^2}{\rho_s l^3}R_T\theta\int_0^l f(x)\,dx - \frac{2\alpha}{r}\theta f(x) - \frac{8\sigma}{r}T_s^3\theta f(x) + \lambda\theta f''_{xx}(x)$$

$$\left(\rho_s = \rho(T_s), \quad R_T = (\partial\ln\rho/\partial T)_{T_s}\right). \qquad (1.3)$$

For homogeneous perturbations (f = 1), (1.3) is put as

$$c\frac{\partial\theta}{\partial t} = -\theta\left[\frac{U^2}{\rho_s l^2}R_T + \frac{2\alpha}{r} + \frac{8\sigma}{r}T_s^3\right]. \qquad (1.4)$$

The solutions to (1.4) are damped exponentials, i.e., the high-temperature state in the heating filament considered as a uniform object is stable.

With spatial perturbations

$$f(x) = \cos(\pi n x/l)$$

the equation for the amplitude $\theta(t)$ is

$$c\frac{\partial\theta}{\partial t} = A\theta, \quad A = \frac{U^2 R_T}{\rho_s l^2} - \frac{2\alpha}{r} - \frac{8\sigma T_s^3}{r} - \frac{\pi^2 n^2\lambda}{l^2}. \qquad (1.5)$$

For A > 0, the amplitude of the perturbations increases exponentially:

$$\theta = \theta_0 \exp(\Omega t), \quad \Omega = A/c.$$

The first mode (n = 1) is the most hazardous, and short-wave perturbations are damped out because of the stabilizing action of the thermal conduction. We see from (1.5) that the physical reason for the instability is the temperature coefficient of resistivity, which under conditions of fixed U may lead to progressive localization of the energy deposition.

The condition A = 0 defines critical values for the characteristics corresponding to failure of the stationary homogeneous solution, which needs to be considered together with (1.2). If T_s is not too high, one can neglect radiation losses, and for a sufficiently long filament ($\ell \gg \sqrt{\pi^2 r \lambda / 2\alpha}$), one can also neglect the thermal-conduction term. Then we get the expressions for the critical filament temperature T_*, voltage U_*, and current density j_* as

$$T_* = T_0 + R_T^{-1}, \quad U_*^2 = 2\rho(T_*)\, l^2 \alpha / r R_T, \quad j_*^2 = 2\alpha / r\rho R_T. \tag{1.6}$$

Here R_T is governed by the temperature dependence of the resistivity. In general, the temperature coefficient R_T is not constant, so the first of the expressions in (1.6) should be considered as an equation for the critical temperature T_*, whose solution exists only for sufficiently rapid increase in $\rho(T)$ (more rapid than linear). With a weaker dependence, the stationary homogeneous state of the current-carrying conductor is absolutely stable. Incorporating the radiation strengthens the requirement for increase in $\rho(T)$ to obtain the runaway. For most metals, $\rho(T)$ is fairly weak over a wide temperature range but sharpens considerably as T approaches the melting point. Typical examples are copper and silver [3]. In that case, the stationary state breaks down and the filament burns out near the melting point of the metal. Substantially lower T_* may occur in semiconductor thermistors with positive temperature coefficients, and also in certain metals. Figure 1 shows R_T for nickel [3]. The intersection between R_T and the $(T - 293)^{-1}$ curve in accordance with (1.6) defines the maximum temperature of the stable homogeneous stationary state: $T_* = 428$ K for $T_0 = 293$ K.

Attempts to heat a nickel filament to $T_s > T_*$ should lead to disruption of the homogeneous temperature distribution and autolocalization of the Joule energy deposition near the point of maximum temperature.

The critical temperature T_* is uniquely determined by the material characteristic R_T only for a long thin filament ($\ell \gg \sqrt{\pi^2 r \lambda / 2\alpha}$). As ℓ decreases, T_* increases because of the stabilizing action of the thermal conduction, which suppresses the temperature inhomogeneities. A sufficiently short conductor ($\ell \ll \sqrt{\pi^2 \lambda r / 2\alpha}$) may be considered as a localized energy deposition object stable against temperature perturbations. If two or more such resistances are joined in series with weak thermal contact, it is readily shown that the system loses its stability on heating: $T_s - T_0 > R_T^{-1}$.

2. For small R_T, the homogeneous high-temperature state is stable ($T_* > T_s$) and the filament lifetime is governed by evaporation. When evaporation is incorporated, (1.1) is supplemented with the kinetic equation

$$\partial r / \partial t = -w(T), \quad w = k_0 \exp(-E/RT), \tag{2.1}$$

in which w is the evaporation rate, k_0 the preexponential factor, E the activation energy, and R the gas constant. The heat loss by evaporation can be neglected in (1.1) because even at high T we find that w is small and correspondingly the energy consumption is negligible by comparison with the electrical power input. The ratio of the thermal relaxation time to the characteristic evaporation time is small, so the evaporation can be considered as quasi-stationary. The quasistationary values of the filament radius $r_s(t)$ and temperature $T_s(r_s)$ are related by (1.2). The heat-transfer coefficient is dependent on the radius ($\alpha = \lambda_1/r$, $\lambda_1 = \mathrm{Nu}\,\lambda_r/2$, with λ_r the thermal conductivity of the surrounding medium and Nu the Nusselt number), with the relationship defined by

$$r_s = \frac{\sigma T_s^4}{U^2/\rho l^2} \left[1 + \sqrt{1 + \frac{2U^2}{\rho l^2} \frac{\lambda_1 (T_s - T_0)}{(\sigma T_s^4)^2}} \right]. \tag{2.2}$$

The (2.1) evaporation kinetic equation governs $r_s(t)$ and $T_s(t)$ with it. The reduction in filament thickness reduces the temperature [the $\rho(T)$ dependence can be neglected for these small R_T]. As T_s falls, so does the evaporation rate, and the retardation in w for large E such as are characteristic of real systems occurs very rapidly. As T_s approaches T_0, w falls almost to zero, so on a real-time scale, the evaporation will continue indefinitely.

This evaporation scenario may not occur because the quasistationary solution is unstable under small perturbations. We seek the nonstationary temperature distribution corresponding
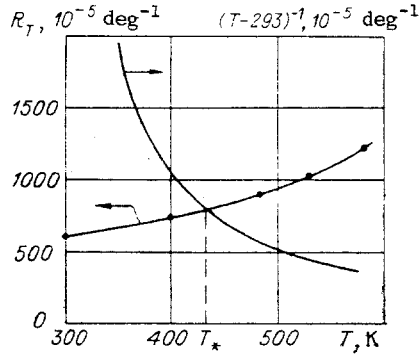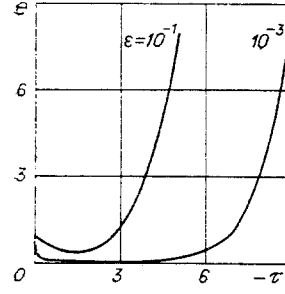
Fig. 1

Fig. 2

to the perturbed solution as the quasistationary component and a small correction dependent on time and the coordinate:

$$T(x, t) = T_s(t) + \theta(t) f(x). \tag{2.3}$$

The perturbation in the radius corresponding to the variation δT is found from (2.1):

$$r = r_s(t) - F(t) f(x), \quad F(t) = \int_0^t (\partial w/\partial T)_{T_s} \theta(t) \, dt. \tag{2.4}$$

We substitute (2.2)-(2.4) into (1.1) and retain only terms of the first order of smallness to get an equation defining the evolution of perturbations in the quasistationary solution:

$$cf(x) \frac{\partial \theta}{\partial t} = \frac{4U^2 F(t)}{l^3 r_s} \left(lf(x) - \int_0^l f(x) \, dx \right) - \frac{4\lambda_1 (T_s - T_0)}{r_s^3} F(t) f(x) -$$

$$- \frac{2\lambda_1}{r_s^2} \theta(t) f(x) - \frac{8\sigma T_s^3}{r_s} \theta(t) f(x) - \frac{2\sigma T_s^4}{r_s^2} F(t) f(x) + \lambda\theta(t) \frac{\partial^2 f}{\partial x^2}. \tag{2.5}$$

For $f(x) = 1$, all the terms on the right are negative, i.e., homogeneous perturbations are damped. For harmonic perturbations,

$$f(x) = \cos(\pi n x/l)$$

(2.5) for $\theta(t)$ becomes

$$c \frac{\partial \theta}{\partial t} = \frac{2}{r_s} \left(\frac{U^2}{\rho l^2} + \frac{\sigma T_s^4}{r_s} \right) F(t) - \theta \left[\frac{2\lambda_1}{r_s^2} + \frac{8\sigma T_s^3}{r_s} + \frac{\pi^2 n^2 \lambda}{l^2} \right]. \tag{2.6}$$

The quasistationary temperature T_s is a single-valued function of time:

$$\frac{dT_s}{dt} = - w(T_s) \left(\frac{dr_s}{dT_s} \right)^{-1}$$

and can be used as an independent variable instead of t. The sharply nonlinear $w(T_s)$ dependence means that the new variable should best be taken as the dimensionless temperature τ measured in the characteristic intervals RT_r^2/E:

$$\tau = (T_s - T_r) E/RT_r^2, \quad T_r = T_s(r_i).$$

The reference temperature T_r corresponds to the initial filament radius r_i, and (2.6) becomes

$$cw(T_r) \frac{E}{RT_r^2 r_s'} \exp(\tau/1 + \beta\tau) \frac{d\theta}{d\tau} = \frac{2}{r_s} \left(\frac{U^2}{\rho l^2} + \frac{\sigma T_s^4}{r_s} \right) \int_0^\tau \left(\frac{T_r}{T_s} \right)^2 r_s' \theta(\tau) \, d\tau +$$

$$+ \theta \left[\frac{2\lambda_1}{r_s^2} + \frac{8\sigma T_s^3}{r_s} + \frac{\pi^2 n^2 \lambda}{l^2} \right], \quad r_s' = \frac{dr_s}{dT_s}, \quad T_s = T_r(1 + \beta\tau), \quad \beta = RT_r/E. \tag{2.7}$$

The evaporation activation energy for a metal is hundreds of kJ per mole, so β is small. For example, $\beta \sim 2 \cdot 10^{-2}$ for tungsten at $T_r = 2 \cdot 10^3$ K. As β is small, we can neglect the changes in all quantities of the order of β in (2.7). A similar technique, including the method of choosing the dimensionless temperature, is used efficiently in the theory of thermal explosion [4]. In the small β approximation $[T_s(\tau) \approx T_r, r_s(\tau) \approx r_i]$, (2.7) becomes

$$\varepsilon \exp(\tau) \frac{d\theta}{d\tau} = \varphi \int_0^\tau \theta \, d\tau + \delta\theta,$$

$$\varepsilon = cw(T_r) \frac{E}{RT_r^2} \frac{r_i^2}{2\lambda_1 r_i'}, \quad \varphi = \frac{r_i'}{r_i}(T_r - T_0)\left[1 + \frac{3\sigma T_r^4 r_i}{\lambda_1(T_r - T_0)}\right], \tag{2.8}$$

$$\delta = 1 + \frac{4\sigma T_r^3 r_i}{\lambda_1} + \frac{\pi^2 n^2 \lambda r_i^2}{2\lambda_1 l^2}, \quad r_i' = (dr_s/dT_s)_{T_r}.$$

Parameters φ and δ (of the order of one) characterize the contributions from the individual forms of heat transfer to the heat balance. The value of ε is defined by the product of three quantities:

$$\varepsilon = \left(\frac{t_r}{t_e}\right)\left(\frac{r_i}{T_r}\frac{1}{r_i'}\right)\beta^{-1}, \quad t_r = \frac{cr_i^2}{2\lambda_1}, \quad t_e = \frac{r_i}{w(T_r)}.$$

The first of the cofactors is the ratio of the thermal relaxation time t_r to the time of complete isothermal evaporation t_e, which is a measure of how far the process is quasistationary. In real situations, this ratio is very small, e.g., the lifetime of a tungsten heating filament is thousands of hours, while the thermal relaxation time is of the order of a second, so ε is close to zero although β^{-1} is large.

That circumstance can be used to construct an approximate solution. We neglect the term in (2.8) proportional to ε to get

$$\theta_1 = a_1 \exp(-\varphi\tau/\delta).$$

That solution applies throughout the range in τ apart from a narrow bounding range near $\tau = 0$, where the derivative $\theta_\tau' \sim \varepsilon^{-1}$. Within the boundary layer, one can neglect the integral term in (2.8), which is proportional to the width of the interval $\Delta\tau \sim \varepsilon$:

$$\theta_2 = a_2 \exp[\delta\varepsilon^{-1}(1 - \exp(-\tau))].$$

We represent the complete solution to (2.8) as the sum of θ_1 and θ_2:

$$\theta = a_1 \exp[-\varphi\tau/\delta] + a_2 \exp[\delta\varepsilon^{-1}(1 - \exp(-\tau))]. \tag{2.9}$$

Equation (2.8) is linear in θ, so we can consider the relative change in θ normalized to the initial value of the perturbation amplitude. Then

$$\theta(0) = 1, \quad a_2 = 1 - a_1.$$

We substitute (2.9) into (2.8) and neglect the less-significant terms in the asymptotic expression ($\varepsilon \to 0$) to get

$$a_1 = \varphi\varepsilon/\delta^2.$$

The numerical solution to (2.8) corresponds well to the approximate one for small ε. Figure 2 shows $\theta(\tau)$ calculated for $\varepsilon = 10^{-1}$–10^{-3}, $\varphi = 1$, $\delta = 1$ (the approximate and exact solutions do not differ on the scale of the figure). In the initial stage, the perturbation amplitude decreases rapidly because of the joint effects of heat transfer and thermal conduction, but subsequently a destabilizing effect occurs from the integral perturbation in the power production, and θ increases without limit.

The instant when the initial value of θ is regained can be taken as the induction period, which defines the filament lifetime t_*. The mean filament temperature during the induction period is reduced by τ_* in the characteristic intervals RT_r^2/E. We put $\theta = 1$ in (2.9) and neglect the second term on the right because ε is small to get

$$\tau_* = \delta\varphi^{-1}\ln(\delta^2/\varphi\varepsilon), \quad T_* = T_r(1 - \beta\tau_*).$$

With φ and δ of the order of one, which corresponds to a long thin filament, the reduction in the mean temperature and thickness preceding the sharpening in the local inhomogeneities is relatively slight even with small ε. As the length l decreases (δ increases), the burn-up of the filament and the drop in T_s after the induction period are increased.

The induction time is calculated from T_* via (2.1):

$$t_* = -\int_{T_r}^{T_*} (r_s'/w) \, dT_s, \quad r_s' = dr_s/dT_s.$$

Asymptotic calculation of the integral for small β gives

$$t_* = t_e r'_i (T_r/r_i) \beta (\delta^2/\varphi \varepsilon)^{\delta/\varphi}, \quad t_e = r_i/w(T_r).$$

The induction period t_* shortens more rapidly as T_r increases than does the isothermal evaporation time t_e:

$$t_* \sim t_e^{1+\delta\varphi^{-1}} \sim \exp\left[E(1 + \delta\varphi^{-1})/RT_r\right].$$

The effective activation energy for the evaporation is $E_* = E(1 + \delta\varphi^{-1})$ and is dependent on the length of the filament, which governs δ. For a fixed T_r, the lifetime increases rapidly as the filament shortens, which is due to the thermal conduction, which suppresses the growth of temperature inhomogeneities in a short filament.

This calculation has been performed with the Thomson thermoelectric effect neglected. To evaluate the parameter range permitting that approximation, we consider the ratio of the quantities $Q_\lambda = \lambda d^2 T/dx^2$ and $Q_S = sjdT/dx$, which govern the power in the dissipative and Thomson sources as appearing in the heat-balance equation (s is the Thomson coefficient, while $j = U/\rho\ell$ is the current density). We take $Q_\lambda \sim \lambda\Delta T/\ell^2$, $Q_S \sim sU\Delta T/\ell^2\rho$ to get $Q_S/Q_\lambda \sim U(s/\rho\lambda)$. Here $s/\rho\lambda$ for conductors does not exceed $5 \cdot 10^{-5}$ V^{-1}, so for U up to the level of several kV, Q_S/Q_λ is negligibly small.

There is experimental evidence for the lack of effect from the Thomson thermoelectric effect from the coincidence between the critical characteristics (T_*, j_*, U_*, t_*) for direct and alternating currents.

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FLOW PAST A SLIM BODY OF REVOLUTION OF A STATIONARY SUPERSONIC FLOW OF A VIBRATIONALLY EXCITED GAS UNDER A SMALL ANGLE OF ATTACK

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The flow past a body under an angle of attack is of interest within the context of the problem of stability of motion of a body in a gas medium. In ordinary gas dynamics the solution of this problem within the slim body approximation is discussed in [1]. The variability of parameters of flow past the body, generated, for example, by nonequilibrium processes in the gas, may substantially affect the aerodynamic characteristics of the body.

In the present study we consider flow past a slim body of revolution of a vibrationally excited gas at a small angle of attack. The solution obtained makes it possible to calculate the transverse force acting on the body, as well as the torque of this force with respect to the tip of the body. It seems that relaxation of vibrational excitation leads to a change in value, and for a sufficient amount of initial nonequilibrium — even a change of sign of the transverse force. The transverse force also acts on a pointed body (without a rounded slice), while in ordinary gas dynamics the linear theory provides a vanishing transverse force [1].

To investigate this problem the symmetry axis of the body of revolution is conveniently chosen to coincide with the x axis, and the stationary supersonic flow, unperturbed by the